place. If we use technology, we can "zoom in" for an accurate estimate of the point of intersection.

Excel has an interesting feature called "Goal Seek" that can be used to find the point of intersection of two lines numerically (rather than graphically). The downloadable Excel tutorial for this section contains detailed instructions on using Goal Seek to find break-even points. ■

Demand and Supply Functions

The demand for a commodity usually goes down as its price goes up. It is traditional to use the letter q for the (quantity of) demand, as measured, for example, in weekly sales. Consider the following example.

Example 3 Linear Demand Function

You run a small supermarket, and must determine how much to charge for Hot'n'Spicy brand baked beans. The following chart shows weekly sales figures for Hot'n'Spicy at two different prices.

Price/Can (p)	$0.50	$0.75
Demand (cans sold/week) (q)	400	350

a. Model the data by expressing the demand q as a linear function of the price p.

b. How do we interpret the slope? The q-intercept?

c. How much should you charge for a can of Hot'n'Spicy beans if you want the demand to increase to 410 cans per week?

Solution

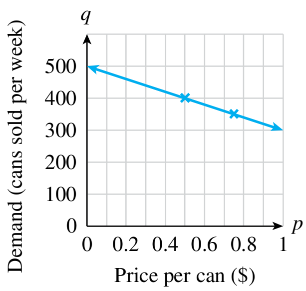

Figure 19

a. A **demand equation** or **demand function** expresses demand q (in this case, the number of cans of beans sold per week) as a function of the unit price p (in this case, price per can). We model the demand using the two points we are given: (0.50, 400) and (0.75, 350).

Point: (0.50, 400)

$$\textbf{\textit{Slope:}}\ m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{350 - 400}{0.75 - 0.50} = \frac{-50}{0.25} = -200$$

Thus, the demand equation is

$$q = mp + b$$
$$= -200p + (400 - (-200)(0.50))$$

or $\quad q = -200p + 500$

Figure 19 shows the data points and the linear model.

b. The key to interpreting the slope, $m = -200$, is to recall (see Example 1) that we measure the slope in units of y per unit of x. In this example, we mean units of q per unit of p, or the number of cans sold per dollar change in the price. Because m is negative, we see that the number of cans sold decreases as the price increases. We conclude that the weekly sales will drop by 200 cans per $1-increase in the price.

To interpret the q-intercept, recall that it gives the q-coordinate when $p = 0$. Hence it is the number of cans the supermarket can "sell" every week if it were to give them away.*

c. If we want the demand to increase to 410 cans per week, we set $q = 410$ and solve for p:

$$410 = -200p + 500$$
$$200p = 90$$
$$p = \frac{90}{200} = \$0.45/\text{can}$$

using *Technology*

A graphing calculator or Excel can be used to find the answer to part (c) numerically; consult the Technology Guides at the end of the chapter to find out how.

* Does this seem realistic? Demand is not always unlimited if items were given away. For instance, campus newspapers are sometimes given away, and yet piles of them are often left untaken. Also see the "Before we go on . . ." discussion at the end of this example.

+ Before we go on...

Q: *Just how reliable* is *the linear model used in Example 3* ?

A: The *actual* demand graph could in principle be obtained by tabulating new sales figures for a large number of different prices. If the resulting points were plotted on the *pq* plane, they would probably suggest a curve and not a straight line. However, if you looked at a small enough portion of this curve, you could closely *approximate* it by a straight line. In other words, *over a small range of values of p, the linear model is accurate.* Linear models of real-world situations are generally reliable only for small ranges of the variables. (This point will come up again in some of the exercises.) ■

Demand Function

A **demand equation** or **demand function** expresses demand q (the number of items demanded) as a function of the unit price p (the price per item). A **linear demand function** has the form

$$q(p) = mp + b$$

Interpretation of *m*

The (usually negative) slope m measures the change in demand per unit change in price. For instance, if p is measured in dollars and q in monthly sales and $m = -400$, then each \$1 increase in the price per item will result in a drop in sales of 400 items per month.

Interpretation of *b*

The y-intercept b gives the demand if the items were given away.

quick Example

If the demand for T-shirts, measured in daily sales, is given by $q(p) = -4p + 90$, where p is the sale price in dollars, then daily sales drop by four T-shirts for every \$1 increase in price. If the T-shirts were given away, the demand would be 90 T-shirts per day.

We have seen that a demand function gives the number of items consumers are willing to buy at a given price, and a higher price generally results in a lower demand. However, as the price rises, suppliers will be more inclined to produce these items (as opposed to spending their time and money on other products), so supply will generally rise. A **supply function** gives q, the number of items suppliers are willing to make available for sale[28], as a function of p, the price per item.

Example 4 Demand, Supply, and Equilibrium Price

Continuing with Example 3, consider the following chart, which shows weekly sales figures (the demand) for Hot'n'Spicy at two different prices, as well as the number of cans per week that you are prepared to place on sale (the supply) at these prices.

Price/Can	\$0.50	\$0.75
Demand (cans sold/week)	400	350
Supply (cans placed on sale/week)	300	500

a. Model these data with linear demand and supply functions.

b. How much should you charge per can of Hot'n'Spicy beans if you want the demand to equal the supply? How many cans will you sell at that price, known as the **equilibrium price?** What happens if you charge more than the equilibrium price? What happens if you charge less?

Solution

a. We have already modeled the demand function in Example 3:

$$q = -200p + 500$$

To model the supply, we use the first and third rows of the table. We are again given two points: (0.50, 300) and (0.75, 500):

Point: (0.50, 300)

$$\textbf{\textit{Slope:}}\ m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{500 - 300}{0.75 - 0.50} = \frac{200}{0.25} = 800$$

So, the supply equation is

$$\begin{aligned} q &= mp + b \\ &= 800p + [300 - (800)(0.50)] \\ &= 800p - 100 \end{aligned}$$

b. To find where the demand equals the supply, we equate the two functions:

$$\begin{aligned} \text{Demand} &= \text{Supply} \\ -200p + 500 &= 800p - 100 \\ -1000p &= -600 \end{aligned}$$

so $$p = \frac{-600}{-1000} = \$0.60$$

This is the equilibrium price. We can find the corresponding demand by substituting 0.60 for p in the demand (or supply) equation.

$$\text{Equilibrium demand} = -200(0.60) + 500 = 380 \text{ cans per week}$$

[28] Although a bit confusing at first, it is traditional to use the same letter q for the quantity of supply and the quantity of demand, particularly when we want to compare them, as in the next example.

So, to balance supply and demand, you should charge \$0.60 per can of Hot'n'Spicy beans and you should place 380 cans on sale each week.

If we graph supply and demand on the same set of axes, we obtain the graphs shown in Figure 20.

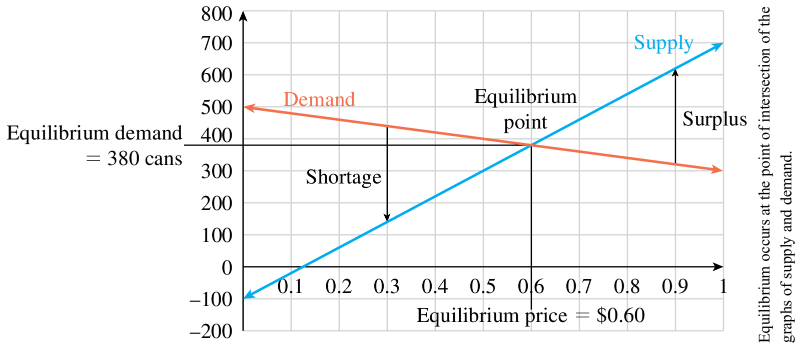

Figure **20**

Figure 20 shows what happens if you charge prices other than the equilibrium price. If you charge, say, \$0.90 per can ($p = 0.90$) then the supply will be larger than demand and there will be a weekly surplus. Similarly, if you charge less—say \$0.30 per can—then the supply will be less than the demand, and there will be a shortage of Hot'n'Spicy beans.

+ *Before we go on...* We just saw in Example 4 that if you charge less than the equilibrium price, there will be a shortage. If you were to raise your price toward the equilibrium, you would sell more items and increase revenue, since it is the supply equation—and not the demand equation—that determines what you can sell below the equilibrium price. On the other hand, if you charge more than the equilibrium price, you will be left with a possibly costly surplus of unsold items (and will want to lower prices to reduce the surplus). Prices tend to move toward the equilibrium, so supply tends to equal demand. When supply equals demand, we say that the market **clears.** ■

Supply Function and Equilibrium Price

A **supply equation** or **supply function** expresses supply q (the number of items a supplier is willing to make available) as a function of the unit price p (the price per item). A **linear supply function** has the form

$$q(p) = mp + b$$

It is usually the case that supply increases as the unit price increases, so m is usually positive.

Demand and supply are said to be in **equilibrium** when demand equals supply. The corresponding values of p and q are called the **equilibrium price** and **equilibrium demand.** To find the equilibrium price, set demand equal to supply and solve for the unit price p. To find the equilibrium demand, evaluate the demand (or supply) function at the equilibrium price.

Change Over Time

Things around us change with time. Thus, there are many quantities, such as your income or the temperature in Honolulu, that it is natural to think of as functions of time.

Example 5 Growth of Sales

The U.S. Air Force's satellite-based Global Positioning System (GPS) allows people with radio receivers to determine their exact location anywhere on earth. The following table shows the estimated total sales of U.S.-made products that use the GPS.*

Year	1994	2000
Sales ($ Billions)	0.8	8.3

a. Use these data to model total sales of GPS-based products as a linear function of time t measured in years since 1994. What is the significance of the slope?

b. Use the model to predict when sales of GPS-based products will reach $13.3 billion, assuming they continue to grow at the same rate.

Solution

a. First, notice that 1994 corresponds to $t = 0$ and 2000 to $t = 6$. Thus, we are given the coordinates of two points on the graph of sales s as a function of time t: (0, 0.8) and (6, 8.3). The slope is

$$m = \frac{s_2 - s_1}{t_2 - t_1} = \frac{8.3 - 0.8}{6 - 0} = \frac{7.5}{6} = 1.25$$

Using the point (0, 0.8), we get

$$\begin{aligned} s &= mt + b \\ &= 1.25t + 0.8 - (1.25)(0) \\ &= 1.25t + 0.8 \end{aligned}$$

Notice that we calculated the slope as the ratio (change in sales)/(change in time). Thus, m is the *rate of change* of sales and is measured in units of sales per unit of time, or billions of dollars per year. In other words, to say that $m = 1.25$ is to say that sales are increasing by $1.25 billion per year.

b. Our model of sales as a function of time is

$$s = 1.25t + 0.8$$

Sales of GPS-based products will reach $13.3 billion when $s = 13.3$, or

$$13.3 = 1.25t + 0.8$$

Solving for t,

$$\begin{aligned} 1.25t &= 13.3 - 0.8 = 12.5 \\ t &= \frac{12.5}{1.25} = 10 \end{aligned}$$

In 2004 ($t = 10$), sales are predicted to reach $13.3 billion.

* Data estimated from published graph. SOURCE: U.S. Global Positioning System Industry Council/*New York Times,* March 5, 1996, p. D1.

Example 6 Velocity

You are driving down the Ohio Turnpike, watching the mileage markers to stay awake. Measuring time in hours after you see the 20-mile marker, you see the following markers each half hour:

Time (h)	0	0.5	1	1.5	2
Marker (mi)	20	47	74	101	128

Find your location s as a function of t, the number of hours you have been driving. (The number s is also called your **position** or **displacement.**)

Solution

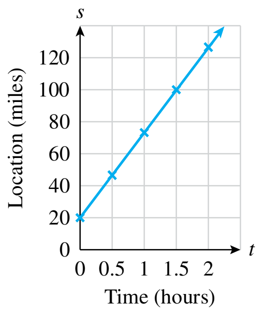

Figure 21

If we plot the location s versus the time t, the five markers listed give us the graph in Figure 21.

These points appear to lie along a straight line. We can verify this by calculating how far you traveled in each half hour. In the first half hour, you traveled $47 - 20 = 27$ miles. In the second half hour you traveled $74 - 47 = 27$ miles also. In fact, you traveled exactly 27 miles each half hour. The points we plotted lie on a straight line that rises 27 units for every 0.5 unit we go to the right, for a slope of $27/0.5 = 54$.

To get the equation of that line, notice that we have the s-intercept, which is the starting marker of 20. From the slope intercept form (using s in place of y and t in place of x) we get:

$$s(t) = 54t + 20$$

Notice the significance of the slope: For every hour you travel, you drive a distance of 54 miles. In other words, you are traveling at a constant velocity of 54 mph. We have uncovered a very important principle:

In the graph of displacement versus time, velocity is given by the slope.

Linear Change Over Time

If a quantity q is a linear function of time t, so that

$$q(t) = mt + b$$

then the slope m measures the **rate of change** of q, and b is the quantity at time $t = 0$, the **initial quantity.** If q represents the position of a moving object, then the rate of change is also called the **velocity.**

Units of m and b

The units of measurement of m are units of q per unit of time; for instance, if q is income in dollars and t is time in years, then the rate of change m is measured in dollars per year.

The units of b are units of q; for instance, if q is income in dollars and t is time in years, then b is measured in dollars.

quick Example

If the accumulated revenue from sales of your video game software is given by $R(t) = 2000t + 500$ dollars, where t is time in years from now, then you have earned \$500 in revenue so far, and the accumulated revenue is increasing at a rate of \$2000 per year.

Examples 1–6 share the following common theme.

General Linear Models

If $y = mx + b$ is a linear model of changing quantities x and y, then the slope m is the rate at which y is increasing per unit increase in x, and the y-intercept b is the value of y that corresponds to $x = 0$.

Units of m and b

The slope m is measured in units of y per unit of x, and the intercept b is measured in units of y.

quick Example

If the number n of spectators at a soccer game is related to the number g of goals your team has scored so far by the equation $n = 20g + 4$, then you can expect 4 spectators if no goals have been scored and 20 additional spectators per additional goal scored.

FAQs What to Use as x and y, and How to Interpret a Linear Model

Q: *In a problem where I must find a linear relationship between two quantities, which quantity do I use as x and which do I use as y?*

A: The key is to decide which of the two quantities is the independent variable, and which is the dependent variable. Then use the independent variable as x and the dependent variable as y. In other words, *y depends on x*.

Here are examples of phrases that convey this information, usually of the form *Find y [dependent variable] in terms of x [independent variable]*:

- Find the cost in terms of the number of items. y = Cost, x = # Items
- How does color depend on wavelength? y = Color, x = Wavelength

If no information is conveyed about which variable is intended to be independent, then you can use whichever is convenient. ■

Q: *How do I interpret a general linear model $y = mx + b$?*

A: The key to interpreting a linear model is to remember the units we use to measure m and b:

The slope m is measured in units of y per unit of x; the intercept b is measured in units of y.

For instance, if $y = 4.3x + 8.1$ and you know that x is measured in feet and y in kilograms, then you can already say, "y is 8.1 kilograms when $x = 0$ feet, and increases at a rate of 4.3 kilograms per foot" without even knowing anything more about the situation! ■

1.4 EXERCISES

● denotes basic skills exercises

◆ denotes challenging exercises

tech Ex indicates exercises that should be solved using technology

Applications

1. ● ***Cost*** A piano manufacturer has a daily fixed cost of \$1200 and a marginal cost of \$1500 per piano. Find the cost $C(x)$ of manufacturing x pianos in one day. Use your function to answer the following questions: *hint* [see Example 1]

   a. On a given day, what is the cost of manufacturing 3 pianos?
   b. What is the cost of manufacturing the 3rd piano that day?
   c. What is the cost of manufacturing the 11th piano that day?

2. ● ***Cost*** The cost of renting tuxes for the Choral Society's formal is \$20 down, plus \$88 per tux. Express the cost C as a function of x, the number of tuxedos rented. Use your function to answer the following questions.

   a. What is the cost of renting 2 tuxes?
   b. What is the cost of the 2nd tux?
   c. What is the cost of the 4098th tux?
   d. What is the marginal cost per tux?