SECTION 2

Measurements in Experiments

SECTION OBJECTIVES

- **List basic SI units and the quantities they describe.**
- **Convert measurements into scientific notation.**
- **Distinguish between *accuracy* and *precision*.**
- **Use significant figures in measurements and calculations.**

NUMBERS AS MEASUREMENTS

Physicists perform experiments to test hypotheses about how changing one variable in a situation affects another variable. An accurate analysis of such experiments requires numerical measurements.

Numerical measurements are different from the numbers used in a mathematics class. In mathematics, a number like 7 can stand alone and be used in equations. In science, measurements are more than just a number. For example, a measurement reported as 7 leads to several questions. What physical quantity is being measured—length, mass, time, or something else? If it is length that is being measured, what units were used for the measurement—meters, feet, inches, miles, or light-years?

The description of *what kind* of physical quantity is represented by a certain measurement is called *dimension.* In the next several chapters, you will encounter three basic dimensions: length, mass, and time. Many other measurements can be expressed in terms of these three dimensions. For example, physical quantities such as force, velocity, energy, volume, and acceleration can all be described as combinations of length, mass, and time. In later chapters, we will need to add two other dimensions to our list, for temperature and for electric current.

The description of *how much* of a physical quantity is represented by a certain numerical measurement depends on the *units* with which the quantity is measured. For example, small distances are more easily measured in millimeters than in kilometers or light-years.

SI is the standard measurement system for science

When scientists do research, they must communicate the results of their experiments with each other and agree on a system of units for their measurements. In 1960, an international committee agreed on a system of standards, such as the standard shown in **Figure 7.** They also agreed on designations for the fundamental quantities needed for measurements. This system of units is called the *Système International d'Unités* (SI). In SI, there are only seven base units. Each base unit describes a single dimension, such as length, mass, or time.

Figure 7
The official standard kilogram mass is a platinum-iridium cylinder kept in a sealed container at the International Bureau of Weights and Measures at Sèvres, France.

Table 2 SI Standards

Unit	Original standard	Current standard
meter (length)	$\frac{1}{10\,000\,000}$ distance from equator to North Pole	the distance traveled by light in a vacuum in $3.33564095 \times 10^{-9}$ s
kilogram (mass)	mass of 0.001 cubic meters of water	the mass of a specific platinum-iridium alloy cylinder
second (time)	$\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)\left(\frac{1}{24}\right) =$ 0.000 011 574 average solar days	9 192 631 770 times the period of a radio wave emitted from a cesium-133 atom

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The units of length, mass, and time are the meter, kilogram, and second, respectively. In most measurements, these units will be abbreviated as m, kg, and s, respectively.

These units are defined by the standards described in **Table 2** and are reproduced so that every meterstick, kilogram mass, and clock in the world is calibrated to give consistent results. We will use SI units throughout this book because they are almost universally accepted in science and industry.

Not every observation can be described using one of these units, but the units can be combined to form derived units. Derived units are formed by combining the seven base units with multiplication or division. For example, speeds are typically expressed in units of meters per second (m/s).

In other cases, it may appear that a new unit that is not one of the base units is being introduced, but often these new units merely serve as shorthand ways to refer to combinations of units. For example, forces and weights are typically measured in units of newtons (N), but a newton is defined as being exactly equivalent to one kilogram multiplied by meters per second squared ($1\text{kg}\bullet\text{m/s}^2$). Derived units, such as newtons, will be explained throughout this book as they are introduced.

SI uses prefixes to accommodate extremes

Physics is a science that describes a broad range of topics and requires a wide range of measurements, from very large to very small. For example, distance measurements can range from the distances between stars (about 100 000 000 000 000 000 m) to the distances between atoms in a solid (0.000 000 001 m). Because these numbers can be extremely difficult to read and write, they are often expressed in powers of 10, such as 1×10^{17} m or 1×10^{-9} m.

Another approach commonly used in SI is to combine the units with prefixes that symbolize certain powers of 10, as illustrated in **Figure 8.**

Did you know?

NIST-F1, an atomic clock at the National Institute of Standards and Technology in Colorado, is one of the most accurate timing devices in the world. NIST-F1 is so accurate that it will not gain or lose a second in nearly 20 million years. As a public service, the Institute broadcasts the time given by NIST-F1 through the Internet, radio stations WWV and WWVB, and satellite signals.

Figure 8
The mass of this mosquito can be expressed several different ways: 1×10^{-5} kg, 0.01 g, or 10 mg.

Table 3
Some Prefixes for Powers of 10 Used with Metric Units

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-18}	atto-	a	10^{-1}	deci-	d
10^{-15}	femto-	f	10^{1}	deka-	da
10^{-12}	pico-	p	10^{3}	kilo-	k
10^{-9}	nano-	n	10^{6}	mega-	M
10^{-6}	micro-	μ (Greek letter *mu*)	10^{9}	giga-	G
			10^{12}	tera-	T
10^{-3}	milli-	m	10^{15}	peta-	P
10^{-2}	centi-	c	10^{18}	exa-	E

Quick Lab

Metric Prefixes

MATERIALS LIST

- balance (0.01 g precision or better)
- 50 sheets of loose-leaf paper

Record the following measurements (with appropriate units and metric prefixes):

- the mass of a single sheet of paper
- the mass of exactly 10 sheets of paper
- the mass of exactly 50 sheets of paper

Use each of these measurements to determine the mass of a single sheet of paper. How many different ways can you express each of these measurements? Use your results to estimate the mass of one ream (500 sheets) of paper. How many ways can you express this mass? Which is the most practical approach? Give reasons for your answer.

The most common prefixes and their symbols are shown in **Table 3.** For example, the length of a housefly, 5×10^{-3} m, is equivalent to 5 millimeters (mm), and the distance of a satellite 8.25×10^{5} m from Earth's surface can be expressed as 825 kilometers (km). A year, which is about 3.2×10^{7} s, can also be expressed as 32 megaseconds (Ms).

Converting a measurement from its prefix form is easy to do. You can build conversion factors from any equivalent relationship, including those in **Table 3.** Just put the quantity on one side of the equation in the numerator and the quantity on the other side in the denominator, as shown below for the case of the conversion 1 mm = 1×10^{-3} m. Because these two quantities are equal, the following equations are also true:

$$\frac{1 \text{ mm}}{10^{-3} \text{ m}} = 1 \quad \text{and} \quad \frac{10^{-3} \text{ m}}{1 \text{ mm}} = 1$$

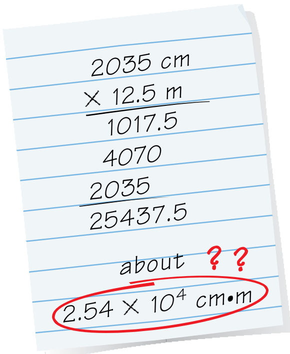
Thus, any measurement multiplied by either one of these fractions will be multiplied by 1. The number and the unit will change, but the quantity described by the measurement will stay the same.

To convert measurements, use the conversion factor that will cancel with the units you are given to provide the units you need, as shown in the example below. Typically, the units to which you are converting should be placed in the numerator. It is useful to cross out units that cancel to help keep track of them.

$$\text{Units } \textit{don't} \text{ cancel: } 37.2 \text{ mm} \times \frac{1 \text{ mm}}{10^{-3} \text{ m}} = 3.72 \times 10^{4} \ \frac{\text{mm}^2}{\text{m}}$$

$$\text{Units } \textit{do} \text{ cancel: } 37.2 \ \cancel{\text{mm}} \times \frac{10^{-3} \text{ m}}{1 \ \cancel{\text{mm}}} = 3.72 \times 10^{-2} \text{ m}$$

(b)



(c)

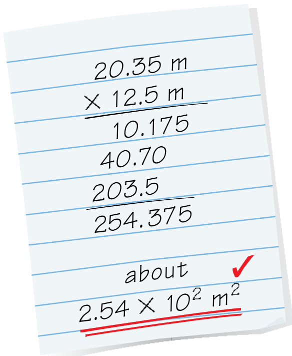


Figure 9
When determining area by multiplying measurements of length and width, be sure the measurements are expressed in the same units.

Both dimension and units must agree

Measurements of physical quantities must be expressed in units that match the dimensions of that quantity. For example, measurements of length cannot be expressed in units of kilograms because units of kilograms describe the dimension of mass. It is very important to be certain that a measurement is expressed in units that refer to the correct dimension. One good technique for avoiding errors in physics is to check the units in an answer to be certain they are appropriate for the dimension of the physical quantity that is being sought in a problem or calculation.

In addition to having the correct dimension, measurements used in calculations should also have the same units. As an example, consider **Figure 9(a),** which shows two people measuring a room to determine the room's area. Suppose one person measures the length in meters and the other person measures the width in centimeters. When the numbers are multiplied to find the area, they will give a difficult-to-interpret answer in units of cm•m, as shown in **Figure 9(b).** On the other hand, if both measurements are made using the same units, the calculated area is much easier to interpret because it is expressed in units of m^2, as shown in **Figure 9(c).** Even if the measurements were made in different units, as in the example above, one unit can be easily converted to the other because centimeters and meters are both units of length. It is also necessary to convert one unit to another when working with units from two different systems, such as meters and feet. In order to avoid confusion, it is better to make the conversion to the same units before doing any more arithmetic.

SAMPLE PROBLEM A

Metric Prefixes

PROBLEM

A typical bacterium has a mass of about 2.0 fg. Express this measurement in terms of grams and kilograms.

SOLUTION

Given: mass = 2.0 fg

Unknown: mass = ? g   mass = ? kg

Build conversion factors from the relationships given in **Table 3.** Two possibilities are shown below.

$$\frac{1 \times 10^{-15}\ \text{g}}{1\ \text{fg}} \text{ and } \frac{1\ \text{fg}}{1 \times 10^{-15}\ \text{g}}$$

Only the first one will cancel the units of femtograms to give units of grams.

$$(2.0\ \cancel{\text{fg}})\left(\frac{1 \times 10^{-15}\ \text{g}}{1\ \cancel{\text{fg}}}\right) = \boxed{2.0 \times 10^{-15}\ \text{g}}$$

Then, take this answer and use a similar process to cancel the units of grams to give units of kilograms.

$$(2.0 \times 10^{-15}\ \cancel{\text{g}})\left(\frac{1\ \text{kg}}{1 \times 10^{3}\ \cancel{\text{g}}}\right) = \boxed{2.0 \times 10^{-18}\ \text{kg}}$$

PRACTICE A

Metric Prefixes

1. A human hair is approximately 50 µm in diameter. Express this diameter in meters.
2. If a radio wave has a period of 1 µs, what is the wave's period in seconds?
3. A hydrogen atom has a diameter of about 10 nm.
   a. Express this diameter in meters.
   b. Express this diameter in millimeters.
   c. Express this diameter in micrometers.
4. The distance between the sun and Earth is about 1.5×10^{11} m. Express this distance with an SI prefix and in kilometers.
5. The average mass of an automobile in the United States is about 1.440×10^{6} g. Express this mass in kilograms.

ACCURACY AND PRECISION

accuracy

a description of how close a measurement is to the correct or accepted value of the quantity measured

precision

the degree of exactness of a measurement

Because theories are based on observation and experiment, careful measurements are very important in physics. But no measurement is perfect. In describing the imperfection, one must consider both a measurement's **accuracy** and a measurement's **precision.** Although these terms are often used interchangeably in everyday speech, they have specific meanings in a scientific discussion. A numeric measure of confidence in a measurement or result is known as *uncertainty.* A lower uncertainty indicates greater confidence. Uncertainties are usually expressed by using statistical methods.

Error in experiments must be minimized

Experimental work is never free of error, but it is important to minimize error in order to obtain accurate results. An error can occur, for example, if a mistake is made in reading an instrument or recording the results. One way to minimize error from human oversight or carelessness is to take repeated measurements to be certain they are consistent.

If some measurements are taken using one method and some are taken using a different method, a type of error called *method error* will result. Method error can be greatly reduced by standardizing the method of taking measurements. For example, when measuring a length with a meterstick, choose a line of sight directly over what is being measured, as shown in **Figure 10(a).** If you are too far to one side, you are likely to overestimate or underestimate the measurement, as shown in **Figure 10(b)** and **(c).**

Another type of error is *instrument error.* If a meterstick or balance is not in good working order, this will introduce error into any measurements made with the device. For this reason, it is important to be careful with lab equipment. Rough handling can damage balances. If a wooden meterstick gets wet, it can warp, making accurate measurements difficult.

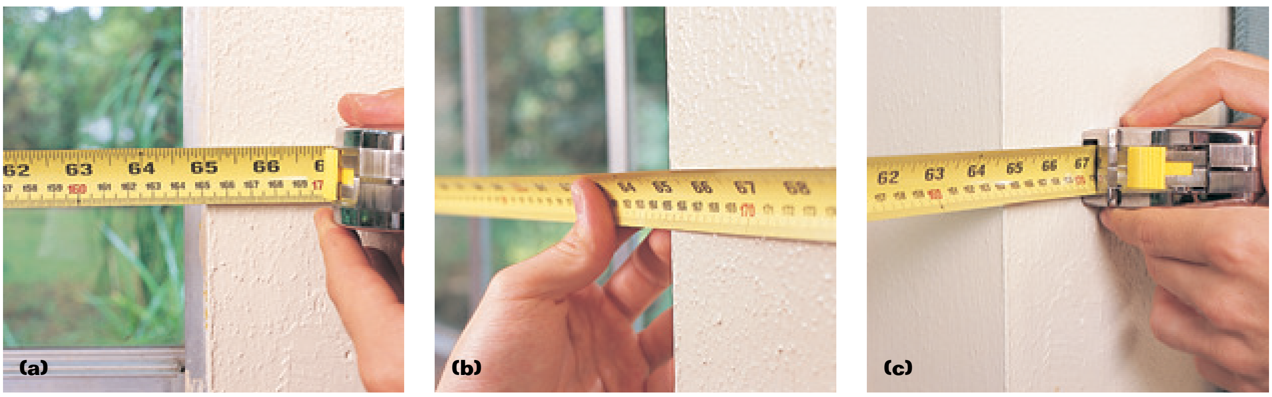


Figure 10
If you measure this window by keeping your line of sight directly over the measurement **(a),** you will find that it is 165.2 cm long. If you do not keep your eye directly above the mark, as in **(b)** and **(c),** you may report a measurement with significant error.

Because the ends of a meterstick can be easily damaged or worn, it is best to minimize instrument error by making measurements with a portion of the scale that is in the middle of the meterstick. Instead of measuring from the end (0 cm), try measuring from the 10 cm line.

Precision describes the limitations of the measuring instrument

Poor accuracy involves errors that can often be corrected. On the other hand, precision describes how exact a measurement can possibly be. For example, a measurement of 1.325 m is more precise than a measurement of 1.3 m. A lack of precision is typically due to limitations of the measuring instrument and is not the result of human error or lack of calibration. For example, if a meterstick is divided only into centimeters, it will be difficult to measure something only a few millimeters thick with it.

In many situations, you can improve the precision of a measurement. This can be done by making a reasonable estimation of where the mark on the instrument would have been. Suppose that in a laboratory experiment you are asked to measure the length of a pencil with a meterstick marked in centimeters, as shown in **Figure 11.** The end of the pencil lies somewhere between 18 cm and 18.5 cm. The length you have actually measured is slightly more than 18 cm. You can make a reasonable estimation of how far between the two marks the end of the pencil is and add a digit to the end of the actual measurement. In this case, the end of the pencil seems to be less than halfway between the two marks, so you would report the measurement as 18.2 cm.

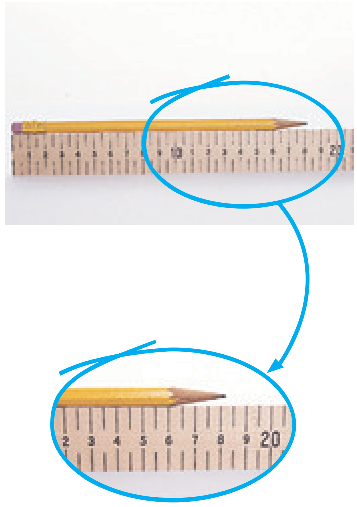


Figure 11
Even though this ruler is marked in only centimeters and half-centimeters, if you estimate, you can use it to report measurements to a precision of a millimeter.

Significant figures help keep track of imprecision

It is important to record the precision of your measurements so that other people can understand and interpret your results. A common convention used in science to indicate precision is known as **significant figures.**

significant figures

those digits in a measurement that are known with certainty plus the first digit that is uncertain

In the case of the measurement of the pencil as about 18.2 cm, the measurement has three significant figures. The significant figures of a measurement include all the digits that are actually measured (18 cm), plus one *estimated* digit. Note that the number of significant figures is determined by the precision of the markings on the measuring scale.

The last digit is reported as a 0.2 (for the estimated 0.2 cm past the 18 cm mark). Because this digit is an estimate, the true value for the measurement is actually somewhere between 18.15 cm and 18.25 cm.

When the last digit in a recorded measurement is a zero, it is difficult to tell whether the zero is there as a place holder or as a significant digit. For example, if a length is recorded as 230 mm, it is impossible to tell whether this number has two or three significant digits. In other words, it can be difficult to know whether the measurement of 230 mm means the measurement is known to be between 225 mm and 235 mm or is known more precisely to be between 229.5 mm and 230.5 mm.

Figure 12
If a mountain's height is known with an uncertainty of 5 m, the addition of 0.20 m of rocks will not appreciably change the height.

One way to solve such problems is to report all values using scientific notation. In scientific notation, the measurement is recorded to a power of 10, and all of the figures given are significant. For example, if the length of 230 cm has two significant figures, it would be recorded in scientific notation as 2.3×10^2 cm. If it has three significant figures, it would be recorded as 2.30×10^2 cm.

Scientific notation is also helpful when the zero in a recorded measurement appears in front of the measured digits. For example, a measurement such as 0.000 15 cm should be expressed in scientific notation as 1.5×10^{-4} cm if it has two significant figures. The three zeros between the decimal point and the digit 1 are not counted as significant figures because they are present only to locate the decimal point and to indicate the order of magnitude. The rules for determining how many significant figures are in a measurement that includes zeros are shown in **Table 4.**

Significant figures in calculations require special rules

In calculations, the number of significant figures in your result depends on the number of significant figures in each measurement. For example, if someone reports that the height of a mountaintop, like the one shown in **Figure 12,** is 1710 m, that implies that its actual height is between 1705 and 1715 m. If another person builds a pile of rocks 0.20 m high on top of the mountain, that would not suddenly make the mountain's new height known accurately enough to be measured as 1710.20 m. The final reported height cannot be more precise than the least precise measurement used to find the answer. Therefore, the reported height should be rounded off to 1710 m even if the pile of rocks is included.

Table 4 Rules for Determining Whether Zeros Are Significant Figures

Rule	Examples
1. Zeros between other nonzero digits are significant.	**a.** 50.3 m has three significant figures. **b.** 3.0025 s has five significant figures.
2. Zeros in front of nonzero digits are not significant.	**a.** 0.892 kg has three significant figures. **b.** 0.0008 ms has one significant figure.
3. Zeros that are at the end of a number and also to the right of the decimal are significant.	**a.** 57.00 g has four significant figures. **b.** 2.000 000 kg has seven significant figures.
4. Zeros at the end of a number but to the left of a decimal are significant if they have been measured or are the first estimated digit; otherwise, they are *not* significant. In this book, they will be treated as *not* significant. (Some books place a bar over a zero at the end of a number to indicate that it is significant. This textbook will use scientific notation for these cases instead.)	**a.** 1000 m may contain from one to four significant figures, depending on the precision of the measurement, but in this book it will be assumed that measurements like this have one significant figure. **b.** 20 m may contain one or two significant figures, but in this book it will be assumed to have one significant figure.

Similar rules apply to multiplication. Suppose that you calculate the area of a room by multiplying the width and length. If the room's dimensions are 4.6 m by 6.7 m, the product of these values would be 30.82 m^2. However, this answer contains four significant figures, which implies that it is more precise than the measurements of the length and width. Because the room could be as small as 4.55 m by 6.65 m or as large as 4.65 m by 6.75 m, the area of the room is known only to be between 30.26 m^2 and 31.39 m^2. The area of the room can have only two significant figures because each measurement has only two. So, the area must be rounded off to 31 m^2. **Table 5** summarizes the two basic rules for determining significant figures when you are performing calculations.

Table 5 Rules for Calculating with Significant Figures

Type of calculation	Rule	Example
addition or subtraction	Given that addition and subtraction take place in columns, round the final answer to the *first column from the left containing an estimated digit.*	$97.3 + 5.85$ $103.15 \xrightarrow{\text{round off}} 103.2$
multiplication or division	The final answer has the same number of significant figures as the measurement having the *smallest* number of *significant figures*.	123×5.35 $658.05 \xrightarrow{\text{round off}} 658$

Calculators do not pay attention to significant figures

When you use a calculator to analyze problems or measurements, you may be able to save time because the calculator can compute faster than you can. However, the calculator does not keep track of significant figures.

Calculators often exaggerate the precision of your final results by returning answers with as many digits as the display can show. To reinforce the correct approach, the answers to the sample problems in this book will always show only the number of significant figures that the measurements justify.

Providing answers with the correct number of significant figures often requires rounding the results of a calculation. The rules listed in **Table 6** on the next page will be used in this book for rounding, and the results of a calculation will be rounded after each type of mathematical operation. For example, the result of a series of multiplications should be rounded using the multiplication/division rule before it is added to another number. Similarly, the sum of several numbers should be rounded according to the addition/subtraction rule before the sum is multiplied by another number. Multiple roundings can increase the error in a calculation, but with this method there is no ambiguity about which rule to apply. You should consult your teacher to find out whether to round this way or to delay rounding until the end of all calculations.

Table 6 Rules for Rounding in Calculations

What to do	When to do it	Examples
round down	• whenever the digit following the last significant figure is a 0, 1, 2, 3, or 4	30.24 becomes 30.2
	• if the last significant figure is an even number and the next digit is a 5, with no other nonzero digits	32.25 becomes 32.2 32.65000 becomes 32.6
round up	• whenever the digit following the last significant figure is a 6, 7, 8, or 9	22.49 becomes 22.5
	• if the digit following the last significant figure is a 5 followed by a nonzero digit	54.7511 becomes 54.8
	• if the last significant figure is an odd number and the next digit is a 5, with no other nonzero digits	54.75 becomes 54.8 79.3500 becomes 79.4

SECTION REVIEW

1. Which SI units would you use for the following measurements?

a. the length of a swimming pool
b. the mass of the water in the pool
c. the time it takes a swimmer to swim a lap

2. Express the following measurements as indicated.

a. 6.20 mg in kilograms
b. 3×10^{-9} s in milliseconds
c. 88.0 km in meters

3. Perform these calculations, following the rules for significant figures.

a. $26 \times 0.02584 = ?$
b. $15.3 \div 1.1 = ?$
c. $782.45 - 3.5328 = ?$
d. $63.258 + 734.2 = ?$

4. Critical Thinking The following students measure the density of a piece of lead three times. The density of lead is actually 11.34 g/cm^3. Considering all of the results, which person's results were accurate? Which were precise? Were any both accurate and precise?

a. Rachel: 11.32 g/cm^3, 11.35 g/cm^3, 11.33 g/cm^3
b. Daniel: 11.43 g/cm^3, 11.44 g/cm^3, 11.42 g/cm^3
c. Leah: 11.55 g/cm^3, 11.34 g/cm^3, 11.04 g/cm^3